# Gauge Symmetry as a Coarse-Grained Limit of Finite-Group Spinor Routing in the SL(4,$\mathbb{C}$) Twistor Framework

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**Abstract**

The global twistor symmetry $SL(4,\mathbb{C}) = \mathrm{Spin}(6,\mathbb{C})$ admits three real forms relevant to four-dimensional physics: the compact form $SU(4)$, the split form $SL(4,\mathbb{R}) = \mathrm{Spin}(3,3)$, and the conformal form $SU(2,2) = \mathrm{Spin}(4,2)$. Each real form yields a distinct but structurally parallel route to particle-physics unification.

In this paper we combine two developments: (1) the $SU(4)$ and $SL(4,\mathbb{R})$ real-form chains of the global twistor symmetry, and (2) Wilson's finite-group quantisation of the electroweak sector via the 288-element subgroup $H_{288} \subset SU(2)_L \times SU(2)_R$. We show that the Standard Model gauge groups arise not as fundamental continuous symmetries, but as coarse-grained limits of discrete spinor-routing dynamics governed by finite subgroups of $SL(4,\mathbb{R})$ and $SL(3,\mathbb{R})$.

Quantum fields emerge as hydrodynamic limits of dense networks of discrete spinor transformations. Gauge bosons appear as coherent waves of routing bias, and curvature (gravitational or gauge) arises from frustration in the routing network. This provides a unified, twistor-based explanation of the emergence of $SU(3) \times SU(2) \times U(1)$, parity violation, flavour mixing, proto-colour, and vacuum-induced gravitation.

## 1 Introduction

Woit emphasises that the complexified conformal group in four dimensions is

$$\mathrm{Spin}(6,\mathbb{C}) = SL(4,\mathbb{C}),$$

with twistor space $T = \mathbb{C}^4$ as a half-spinor representation. The real forms

$$SU(4), \qquad SL(4,\mathbb{R}), \qquad SU(2,2)$$

provide three parallel routes to unification.

In earlier work [1], we showed that the compact form $SU(4)$ yields the Standard Model gauge structure via

$$SU(4) \supset SU(3) \times U(1)_G \supset SU(2) \times U(1)_Y.$$

Wilson's programme instead begins with the split real form

$$SL(4,\mathbb{R}) = \mathrm{Spin}(3,3),$$

whose maximal compact subgroup

$$SO(4) \simeq SU(2)_L \times SU(2)_R$$

naturally encodes chiral electroweak structure. In [**?**], we constructed Wilson's proposed finite subgroup

$$H_{288} \subset SU(2)_L \times SU(2)_R \subset SO(4) \subset SL(4,\mathbb{R}),$$

with normal core $Q_8^{(L)} \times Q_8^{(R)}$ and quotient $S_3 \times Z_3$.

The purpose of this paper is to show that continuous gauge groups arise as coarse-grained limits of such finite-group spinor-routing dynamics.

## 2 Finite-Group Spinor Routing

At each spacetime point we consider a local $SL(4,\mathbb{R})$ twistor-spinor

$$\Psi = (\psi_{L,1}, \psi_{L,2}, \psi_{R,1}, \psi_{R,2})^T,$$

with left and right doublets acted upon by $SU(2)_L$ and $SU(2)_R$.

Wilson's finite subgroup $H_{288}$ consists of:

- a quaternionic core $Q_8^{(L)} \times Q_8^{(R)}$ encoding discrete spin and chirality flips,
- a broken left triplet $G_L$ encoding weak-isospin asymmetry,
- an unbroken right triplet $2T_R$ encoding right-handed flavour structure,
- an outer quotient $S_3 \times Z_3$ encoding flavour permutations and proto-colour phases.

These elements act as discrete routing operations on the spinor components. For example, the weak isospin raising operator $d_L \to u_L$ is realised by a finite rotation

$$R_q = \exp\left(\frac{\pi}{2} L_1\right),$$

while the lepton lowering operator $\nu_{eL} \to e_L^-$ is realised by

$$R_\ell = \exp\left(-\frac{\pi}{2} L_1\right).$$

A beta-decay event is represented by the word

$$\mathcal{B} = R_q R_\ell C_\nu,$$

where $C_\nu \in Q_8^{(R)}$ routes a virtual right-handed neutrino into an anti-neutrino.

## 3 Emergence of Continuous Gauge Symmetry

The continuous Lie algebras $su(2)_L$ and $su(3)_c$ arise as Lie closures of finite subgroups.

### 3.1 Emergent $SU(2)_L$

The infinitesimal generators

$$\tilde{T}_+ = L_1, \qquad \tilde{T}_- = -L_1, \qquad \tilde{T}_3 = L_3$$

satisfy the $su(2)$ commutation relations. These are precisely the infinitesimal versions of the discrete moves $R_q$ and $R_\ell$.

Thus:

$$SU(2)_L = \overline{\langle R_q, R_\ell \rangle},$$

the closure of the finite subgroup generated by weak-isospin raising and lowering moves.

### 3.2 Emergent $SU(3)_c$

Proto-colour is encoded in the outer quotient $S_3 \times Z_3$. The discrete generators

$$C_\pi = \text{3-cycle}, \qquad C_\phi = \text{phase rotation}$$

embed into $SU(3)$ as finite rotations. Their Lie closure yields the full colour algebra:

$$SU(3)_c = \overline{\langle C_\pi, C_\phi \rangle}.$$

### 3.3 Gauge Fields as Routing Bias

A gauge field is the continuum limit of a bias in the probabilities of applying certain discrete routing moves. A $W^-$ boson is a coherent wave of enhanced probability for the word $R_q R_\ell C_\nu$ across neighbouring vacuum cells.

Curvature (gauge or gravitational) is frustration in the routing network: the failure of spinor transport around a loop to return to its initial state.

## 4 Gravity from SL(4,$\mathbb{R}$) Routing

Vacuum polarisation defines an effective metric $g^{\text{eff}}_{\mu\nu}$ as a coarse-grained description of routing biases in the $SL(4,\mathbb{R})$ network. The Lorentz group $SO(3,1)$ emerges from the symmetry-breaking pattern

$$SL(4,\mathbb{R}) \supset SO(3,3) \supset SO(3,1).$$

The Abelian factor $U(1)_G$ associated with the diagonal generator of $SL(4,\mathbb{R})$ yields both gravitational attraction and cosmological repulsion, with Newton's constant

$$G = \alpha\, \rho_G$$

proportional to the vacuum energy density.

## 5 Twistor Interpretation

The conformal real form $SU(2,2)$ provides a twistor description of null routing. The chain

$$SU(2,2) \subset SL(4,\mathbb{C}) \supset SL(4,\mathbb{R}),\ SU(4)$$

shows that all three real forms are different projections of the same underlying twistor-spinor dynamics.

The finite subgroup $H_{288}$ is a discrete twistor structure for the $SL(4,\mathbb{R})$ branch.

## 6 Conclusion

We have shown that:

- The Standard Model gauge groups are not fundamental.
- They arise as coarse-grained limits of finite-group spinor-routing dynamics.
- Wilson's $H_{288}$ subgroup provides the discrete electroweak skeleton.
- Proto-colour $S_3 \times Z_3$ provides the discrete colour skeleton.
- Gravity arises from routing biases in the $SL(4,\mathbb{R})$ network.
- All three real forms of $SL(4,\mathbb{C})$ provide parallel unification routes.

This yields a mechanistic, twistor-based unification in which quantum fields are emergent hydrodynamic limits of discrete spinor dynamics.

## References

[1] N. B. Cook, *Unification by SL(4,C) Twistors*, viXra:2608.0002 (2026).